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AB1/BC1) a) G'(5)=-24.58750937 tons per hour^2

At t=5 hours, the rate at which unprocessed gravel is arriving at the plant is DECREASING at a rate of 24.5875 tons per hour^2.

b) total amount = $\int_0^8 G(t)dt = 825.5510886$ tons arrive during the course of the day

c) $G(5) = 98.14076403$ tons per hour. The amount of unprocessed gravel at the plant is decreasing at t=5 hours because the rate of change of unprocessed gravel arriving at the plant is 98.1407 tons per hour and the unprocessed gravel is being processed at the rate of 100 tons per hour (which is faster than the arrival rate of 98.1407 tons per hour).

d) The rate of change of unprocessed gravel $U(t) = G(t) - 100$. A relative maximum would require $U(t)$=0 (it is never undefined).

This happens when $t = 4.92348$ hours.

$\int_0^{4.92348} G(t)dt + 500 - 100(4.92348) = 635.3761231$ tons. There are only 500 tons at t=0 and 525.5510886 tons at t=8 hours.

AB2) a) For the speed to be 2, the velocity must be 2 or -2.

$v(t) = 2 \Rightarrow t = .94369687, 3.1276299$

$v(t) = -2 \Rightarrow t = 3.473402$

In the interval 2<t<4, there are two values of *t* for which the speed is 0:

t=3.1276 and *t*=3.4734

b) $s(t) = 10 + \int_0^t v(x)dx = 10 + \int_0^t -2 + (x^2 + 3x)^{6/5} - x^3 dx$

$s(5) = 10 + \int_0^5 -2 + (x^2 + 3x)^{6/5} - x^3 dx = -9.207329838$

c) $v(t)$ must be 0 (or undefined, but this function is always defined) for the particle to change direction.

$v(t) = 0 \Rightarrow t = 0.53603315, 3.3177563$

$v'(0.53603315) = 4.691132259 > 0$, so the position function is concave up and the position must be a minimum (and so the velocity changes direction)

$v'(3.3177563) = -11.77356831 < 0$, so the position function is concave down and the position must be a maximum (and so the velocity changes direction)

d) $\begin{array}{l} v(4) = -11.47575789 \\ a(4) = -22.29571535 \end{array}$. Since both velocity and acceleration are negative, they "pull in the same direction" and so the particle is speeding up (the speed is increasing).

AB3/BC3) a) $C'(3.5) \approx \frac{C(4) - C(3)}{4-3} = \frac{12.8 - 11.2}{1} = 1.6$ ounces per minute

b) *C* is differentiable and therefore also continuous. So the MVT will apply to this situation. $\frac{C(4) - C(2)}{4-2} = 2$ ounces per minute, and so there is a *c* in (2,4) for which $C'(2) = 2$. This is what we are to show.

c)
$\frac{1}{6}\int_0^6 C(t)dt \approx \frac{1}{6}\left(C(1)*(2-0)\right)+C(3)*(4-2)+C(5)*(6-4)) = \left(\frac{1}{6}\right)(2)(5.3+11.2+13.8)$ ounces. $\frac{1}{6}\int_0^6 C(t)dt$ is the average value of *C(t)* – in other words, it is the average number of ounces of coffee in the coffee cup over the interval from t=0 minutes to t=6 minutes.

d) The rate at which the amount of coffee in the cup is changing is $B'(5) = (16)(.04)e^{-0.4(5)}$ ounces per minute.

AB4/BC4) a) A function has a local minimum at $x = a$ if $x = a$ is a **critical point** ($f'(a) = 0$ or $f'(x)$ is undefined at *a*) **AND** $f'(x)$ changes sign at $x = a$. We have a graph of $f'(x)$ and this question is easy:

$x = 1$: $f'(x)$ is 0 but doesn't change sign

$x = 4$: $f'(x)$ is 0 and changes from positive to negative. So $f(x)$ changes from increasing to decreasing there, and we have a local maximum.

$x = 6$: $f'(x)$ is 0 and changes from negative to positive. So $f(x)$ changes from decreasing to increasing there, and we have our **only local minimum of *f*.**

b) The absolute minimum value will be at $x = 0$.

$f(8) = 4$: given

$\int_6^8 f'(x)dx = f(8) - f(6) = 4 - f(6) = 7$. (the area under that part of the curve is given). So $f(6) = -3$

$\int_0^6 f'(x)dx = f(6) - f(0) = -3 - f(0) = 2 + 6 - 3 = 5$. Again the area is given. So $f(0) = -8$

The absolute minimum occurs at an endpoint or a relative minimum, and so the absolute minimum is -8 at $x = 0$.

c) To be concave down, the derivative will be decreasing. To be increasing, the derivative will be positive.

Answer: $(0,1) \cup (3,4)$

d) $g'(x) = 3\left(f(x)\right)^2 f'(x)$ by the chain rule. So $g'(x) = 3\left(-\frac{5}{2}\right)^2(4) = 75$. This is the slope of the line tangent to *g* at $x = 3$.

AB5) a) area of R =

$$\int_0^2 \left(4\cos\left(\frac{\pi x}{4}\right)\right) - \left(2x^2 - 6x + 4\right)dx = \left(\frac{16}{\pi}\sin\left(\frac{\pi x}{4}\right) - \frac{2x^3}{3} + 3x^2 - 4x\right)\Big|_0^2$$

This is $\left(\frac{16}{\pi}\sin\left(\frac{\pi}{2}\right) - \frac{16}{3} + 12 - 8\right) - 0 = \frac{16}{\pi} - \frac{4}{3}$

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b) volume = $\pi\int_0^2 \left(4-\left(2x^2-6x+4\right)\right)^2 - \left(4-4\cos\left(\frac{\pi x}{4}\right)\right)^2 dx$

c) volume = $\int_0^2 \left(\left(4\cos\left(\frac{\pi x}{4}\right)\right)-\left(2x^2-6x+4\right)\right)^2 dx$

AB6) a) $\frac{dy}{dx} = e^y\left(3x^2-6x\right)\Big|_{(1,0)} = -3$ so the tangent line is: $y-0=-3(x-1)$.

$y=-3(1.2-1)=-.6$, so $f(1.2)\approx -0.6$.

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b)
$$e^{-y}dy = (3x^2-6x)dx$$
$$-e^{-y} = x^3-3x^2+C$$
$$at\,(1,0) \Rightarrow -1=-2+C \Rightarrow C=1$$
$$-e^{-y} = x^3-3x^2+1 \Rightarrow y=-\left(\ln\left(-x^3+3x^2-1\right)\right)$$

BC2) a) area of S = area of the sector from $\theta=\frac{5\pi}{6}$ all the way around to $\theta=\frac{\pi}{6}$ + area (between $\theta=\frac{\pi}{6}$ and $\theta=\frac{5\pi}{6}$) of the 'sector' bounded by the curve $r=4-2\sin\theta$

So, area = $\pi(3)^2\left(\frac{4\pi/3}{2\pi}\right)=\frac{18\pi}{3}=6\pi + \int_{\pi/6}^{5\pi/6}\frac{1}{2}(4-2\sin\theta)^2\,d\theta = 24.70873079$

b) $x(\theta)=r\cos(\theta)=\left(4-2\sin(\theta)\right)\left(\cos(\theta)\right)=4\cos(\theta)-2\sin(\theta)\cos(\theta)$

$x(t)=4\cos\left(t^2\right)-2\sin\left(t^2\right)\cos\left(t^2\right)$

Find the zeros of $4\cos\left(t^2\right)-2\sin\left(t^2\right)\cos\left(t^2\right)-(-1)$ and get $t=1.4279793$

c) Similarly to (b), find $y(t)$.

$\vec{s}(t) = <4\cos\left(t^2\right)-2\sin\left(t^2\right)\cos\left(t^2\right), 4\sin\left(t^2\right)-2\sin^2\left(t^2\right)>$

$\vec{v}(1.5) = <\frac{d}{dt}x(1.5), \frac{d}{dt}y(1.5)> = <-8.072101342, -1.672933458>$

BC5) a) $\lim_{x\to 0}\frac{f(x)+1}{\sin(x)}$ is indeterminate of the form $\frac{0}{0}$, so I can use L'Hospital's rule.

$\lim_{x\to 0}\frac{f(x)+1}{\sin(x)} = \lim_{x\to 0}\frac{f'(x)}{\cos(x)} = \frac{2}{1} = 2$

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b)

$$x = 0; y = -1 \Rightarrow \frac{dy}{dx} = 2$$

$$x = 0 + \frac{1}{4} \Rightarrow \Delta y \approx 2\left(\frac{1}{4}\right) = \frac{1}{2} \Rightarrow y \approx -1 + \frac{1}{2} = -\frac{1}{2}$$

$$\left.\frac{dy}{dx}\right|_{(1/4,-1/2)} = \left(\frac{1}{4}\right)\left(2\left(\frac{1}{4}\right)+2\right) = \frac{5}{8}$$

$$x = \frac{1}{4} + \frac{1}{4} \Rightarrow \Delta y \approx \left(\frac{5}{8}\right)\left(\frac{1}{4}\right) = \frac{5}{32} \Rightarrow y \approx -\frac{1}{2} + \frac{5}{32} = -\frac{11}{32}$$

Therefore, $f\left(\frac{1}{2}\right) \approx -\frac{11}{32}$

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c)

$$\frac{1}{y^2} dy = \left(2x+2\right) dx$$

$$\frac{-1}{y} = x^2 + 2x + C$$

$$at\,(0,-1)$$

$$1 = C \Rightarrow \frac{-1}{y} = x^2 + 2x + 1 \Rightarrow y = \frac{-1}{\left(x+1\right)^2}$$

BC6) a) $P_1\left(x\right) = f(0) + f'(0)x = -4 + f'(0)x$

So $P_1\left(\frac{1}{2}\right) = -4 + f'(0)\left(\frac{1}{2}\right) = -3 \Rightarrow f'(0)\left(\frac{1}{2}\right) = +1 \Rightarrow f'(0) = 2$

b) $P_3(x) = -4 + 2x - \frac{2x^2}{3 \cdot 2!} + \frac{x^3}{3 \cdot 3!}$

c) $h(0) = 7; h'(0) = f(2\cdot 0) = -4; h''(0) = 2f'(2\cdot 0) = 4; h'''(0) = 4f''(2\cdot 0) = -\frac{8}{3}$

so $H_3(x) = 7 - 4x + \frac{4x^2}{2!} - \frac{8x^3}{3 \cdot 3!}$